INDEX OF NETWORK RESILIENCE (INR) FOR URBAN WATER DISTRIBUTION SYSTEMS

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Abstract: A unique demographic shift towards urban centres has necessitated incorporation of sustainability principles in the tenets of urban infrastructure planning and design. Adopting resilience as the indicator of sustainability, this paper presents a novel index of network resilience (INR) for urban water distribution systems. The index developed in this paper incorporates six network attributes to develop a composite INR based on the topology of the water distribution systems. A Multi-Criteria Analysis (MCA) using the Weighted Summation approach is employed to evaluate the alternative configurations which would satisfy the demand and other hydraulic requirements. Analytic Hierarchy Process (AHP) to assign weights to the attributes and has been optimized for two scenarios: resilience and efficiency. Using the original configuration of Anytown network as the base case scenario, four alternative designs were developed. The comparative performance results are presented herein the paper.

Keywords: resilience index; urban infrastructure; complex networks; urban water systems; urban sustainability; multi-criteria analysis

INTRODUCTION

Amid an unprecedented rise in global urban population in recent decades, provision of sustainable urban infrastructure is one of the key challenges to urban planners, socioeconomic decision makers and engineers alike. With more than a billion people in the lesser developed regions not having access to safe drinking water and developed countries facing mounting challenge to cope up with the aging infrastructure, provision of potable water is of particular importance. Despite several research efforts, there is lack of a comprehensive indicator of sustainability, in particular for infrastructure systems. Resilience, or the capacity of a system to absorb shocks and perform under perturbations, can serve as an appropriate indicator of sustainability. Water distribution systems (WDS) are often vulnerable to natural disasters, like earthquakes and are potential targets for terrorist attacks. Better comprehension about the resilience of these systems would allow us to identify and address these vulnerabilities.

In this article, we report a novel resilience index for urban water distribution systems based on the network structure of the system. Water distribution systems being complex networks, a graph theoretical network analysis reveals several attributes related to the efficiency and resilience of the system. Six metrics are identified to quantify the efficiency, robustness and path redundancy of the network system. A Multi-Criteria Analysis (MCA) approach is employed to evaluate the alternative configurations which satisfy the hydraulic requirements of a network. An Analytic Hierarchy Process (AHP) is used to assign weights to the aforementioned attributes as it would allow tailoring the weights according to the requirements of the stakeholders and to cater to the particular geographic and demographic choices. Two different weighting scenarios are used for this purpose, one where all the metrics are assigned the weights to maximize the flow efficiency of flow in the network and the other where the weights are preferentially assigned to maximize the resilience of the network, to identify the possibility of potential trade-off between efficiency and resilience according to the layout of a distribution network. The Anytown Network (AtN) (1) has been used as case-study for this exercise. Using the hypothetical AtN the base-case scenario, four alternate configurations have been developed that satisfy the water demands. These four alternatives have then been compared amongst each other and with the original configuration to assess their resilience based on their network topology. Finally, we discuss the results, limitations of the present methodology and future direction of this research.

URBAN INFRASTRUCTURE SYSTEMS AND COMPLEX NETWORKS

Research in the genre of complex networks have increased in the recent years. Complex networks are usually characterized by a distributed system consisting of multiple interconnected components in a nontrivial configuration in which the function is reliant on the network structure or topology (2). The interplay between network structure and their functionality is evident from the structure and functional attributes of the two major classes of complex networks: scale-free and random. The scale-free networks are characterized by their non-uniform degree distribution which follows a power law, i.e. there a few hubs with a lot of connections and majority of the nodes have a low degree of connectivity. Random networks, conversely, have a degree distribution which adheres to a uniform bell-curve, i.e. majority of the nodes have approximately the same degree of connectivity. This topological difference considerably has an impact on their resiliency. While scale-free networks are more resilient than random networks in case of accidental perturbations, they are more vulnerable than their counterpart in the event of targeted attack at the hubs.

The ubiquity and importance of complex networks as the core structural framework of numerous technological and societal systems has garnered significant research interest to comprehend the dynamics of the network formation and growth in the recent past and has led into efforts to analyze the vulnerability and resilience of these networks against natural or anthropogenic perturbations. While there has been considerable research involving complex network in other infrastructure sectors (3–6) or even then urban system in its entirety (7), application of complex networks to analyze urban water systems is relatively sparse (8), (9). Water distribution systems are spatially distributed systems where multiple components are connected by physical links. In a graphical representation of the water distribution networks, pipes and other connections represents the edges of the graph while demand and supply points are the nodes in the graph. The complexity of the resultant graph results from the complex interactions between the different components, uncertainty and nontrivial configuration of the system (8). The complexity and uncertainty arises from the wide range of possible combinations of pipe sizes, pipe materials, connectivity layouts, spatial location of the demand and supply nodes and variability and uncertainty in water demand which are not captured in the traditional engineering optimization methods. Traditionally urban water systems – distribution systems in particular – have been optimized with respect the capital cost. Consequently, analytical and simulation techniques abounded including linear programming (10), non-linear programming (11), integer goal programming (12) and Monte Carlo simulations (13), which have been employed depending on the scale and complexity of the problem. With the advancement in computation abilities, multi-criteria decision making (MCDM) analyses have been employed to optimize urban water-distribution systems through the use of genetic algorithms and other criterion like reliability of supply has been incorporated in the decision-making (14–16).

To elucidate further, the following optimization paradox for water utilities can be considered. As discussed previously, utilities try to optimize the efficiency of water delivery while minimizing the cost. From the network perspective, while the former condition requires a star-shaped network the latter demands a tree-structure (Figure 1) and both scenarios would perform disastrously in the event of any disturbance. Interestingly, the two networks in discussion would perform radically different if the water quality at the consumer's faucet parameter is considered. The star network would perform best due to the low residence time of treated water in the distribution system and would be egalitarian to its consumers as all of them would receive the water having approximately the same residence time and quality, because the reactions with chlorine with background organic matter would have longer to form by products and the water would have more of an opportunity to leach constituents (corrosion products and others) from the pipe network. Conversely, in case of the tree structure, the consumers closest to the source would receive the best quality water and those further done the tree would receive the worst quality water. Complex network approaches can overcome these challenges encountered in design and operation, while increasing the reliability of the system.

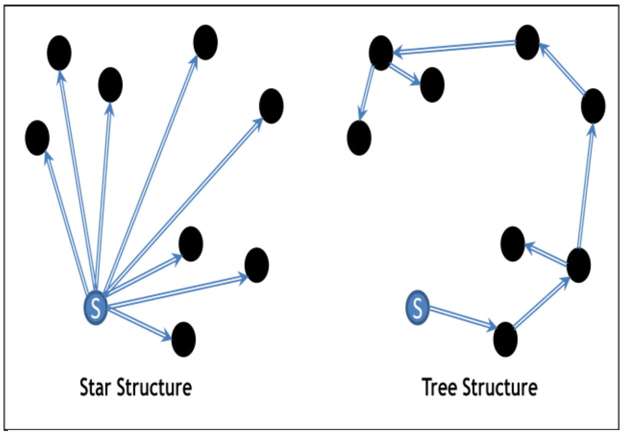

Figure 1: Schematic depicting star and tree structure network for identical spatial set of source and demand nodes.

In general the structure of water distribution systems is correlated with the local topography and spatial distribution of the demand. In the node-link configuration of water distribution systems, the nodes can be distinctly grouped into source nodes, control and distribution nodes and demand nodes. On the other hand, the links are represented by the transmission and distribution pipes with different length, size and other physical attributes dependent on the material of the pipes. Essentially, the water distribution systems are directed graphs, i.e. the flow within the system is directional. However, the direction of flow in the distribution system alternates (except for those directly connected to the source or sinks) occasionally depending on operational flow and pressure requirements, rendering the consideration of water distribution systems as weighted digraphs exceedingly complex computationally. In addition, a comprehensive assessment for resilience of water distribution networks should also incorporate the nontopological specifications of the network, which include the size and material of the links and importance of the nodes. Adoption of such an approach, though would reveal a more realistic correlation between the topology of the network and the operational aspects related to the analysis of reliability of the network; the approach would require significant analysis of empirical pressure and flow data coupled with extensive simulation scenarios making the process too costly computationally (8). Thus, this study treats the water distribution networks as undirected graphs and is based on the statistical properties of the network topology using graph theory to identify and recognize the structural patterns and building blocks of the networks. While this approach might not be able to assess the vulnerabilities completely it would provide a reasonable comprehension about the network structure and would be able to predict critical vulnerabilities associated with the structure of the network.

ANYTOWN NETWORK (ATN)

The Anytown network is a hypothetical network for a hypothetical Anytown, US. It has been used as the case-study in numerous studies about the optimization of using a suite of genetic algorithms (17), to determine the trade-off between reliability and total cost (15), entropy based design (18), to name a few. The original distribution system as proposed is shown in Figure 2 (19). The town gets its water from a single source which is treated at a single centralized treatment plant. In addition, two overhead reservoirs are used in the system for storage purposes. The original configuration has 22 nodes, inclusive of the source and the reservoirs and 41 links or connections between the nodes.

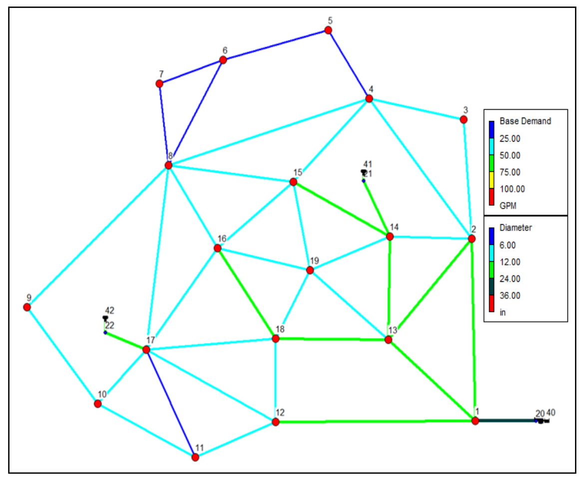

Figure 2: Structure of Anytown Network

Consequentially, all the alternate configurations have the same number of nodes, but they vary in the number of connection or links. The alternate configurations would be termed AtN^1, AtN^2, AtN^3 and AtN^4 in this communication for the sake of brevity.

THE FRAMEWORK FOR INDEX OF NETWORK RESILIENCE (INR)

Network Metrics

This study incorporates six metrics of the network structure, ① graph diameter *(d)*, ② characteristic path-length *(l)*, ③ central-point dominance *(c'_b)*, ④ critical ratio of defragmentation *(f_c)*, ⑤ algebraic connectivity *(λ_2)*, and ⑥ meshedness coefficient *(r_m)*; to develop a composite INR. The first two attributes are related to the efficiency of the system, the third reflects the dominance of a particular node in maintaining the integrity of the network and the last three are surrogate measures of the robustness and path redundancy of the network to failure of one or more nodes or links.

In the graphical representation, any water distribution can be modeled as a mathematical planar graph $G = G(N, M)$, where N is the set of n nodes and M is the set of m edges in the graph. The efficiency of flow of water, or any other resource or information, per se, in a graph is characterized by the geodesic path length, i.e. the number of edges it has to traverse to reach from any node n_i to $n_{j, j \neq i}$ in an undirected connected path. The graph diameter *(d)* is a measure of the maximum graph eccentricity defined as the maximum value of the shortest geodesic paths. The characteristic path length is the average of the shortest path-lengths in a graph, i.e. the average degree of separation between all nodes of the graph and can be obtained through the following expression:

$$l = \frac{1}{n(n-1)} \sum_{(i \neq j)} d_{ij} \qquad (1)$$

where, d_{ij} is the shortest geodesic path between nodes n_i to $n_{j, j \neq i}$ in an undirected connected path and n is the number of nodes in the graph. A shorter characteristic path length and a smaller graph diameter would indicate a more efficient network. A critical measure of the structural organization of the network can be obtained through the central-point dominance (c'_b) metric which indicates the dominance of the central point(s) of the network in regulating the flow within the network or the degree of concentration of the network around the central point(s). c'_b can be defined as the average difference in betweenness centrality between the most central point having the maximum value of betweenness and all others (20). c'_b can be determined through Equation 2.

$$c'_b = \frac{\sum_{i=1}^{n} \left[c_b\left(n_k^*\right) - c_b\left(n_i\right) \right]}{n-1} \qquad (2)$$

where, $c_b(n^*_k)$is the maximum relative betweenness centrality value around the central node k, $c_b(n_i)$is the relative betweenness centrality value for any node i in the network and n is the total number of nodes. The betweenness centrality of node k can be obtained from Eqn. 3.

$$C_b(k) = \sum_{s \neq k \neq t \in V} \frac{\sigma_{st}(k)}{\sigma_{st}} \qquad (3)$$

where, σ_{st} is the total number of shortest paths from node s to node t and $\sigma_{st}(k)$ is the number of those paths passing through node k. As implied from the summation indices, the betweenness centrality of a node scales with the number of pair of nodes. Thus this value may be normalized to a relative betweenness centrality value (c'_b) by dividing C_b with the number of pairs of nodes not including k, so that the lower and upper bound of the value of c'_b are 0 and 1. The value is 0 for graphs where all nodes have the same centrality value and is 1 for only star or wheel graphs.[1] While graph diameter and characteristic path length are explicitly related to the efficiency of the network, central point dominance is related to both the efficiency and resilience aspect of the network configuration. While a higher value of the central point dominance will be more economical and would facilitate flow in the network, simultaneously it would make the network more vulnerable compromising its resilience due to the high sensitivity of the network around the central point(s). Water distribution systems are inherently vulnerable to attacks at or removal of

[1] In a star graph all nodes have a degree of one except for one node which has a degree equal to the sum of all other nodes, i.e. in a star graph with 5 nodes and 4 links, 4 nodes have a degree of 1 each while the 5th node has a degree of (1+1+1+1) 4. On the other hand, a wheel graph is a graph where all nodes have a degree of 3, except one, which has a degree of *(n-1)*, where *n* is the number of nodes in the graph.

certain nodes like reservoirs and tanks, i.e. the removal of these nodes renders the system nonfunctional. A higher value of c'_b indicates presence of more critical nodes in the network, the removal of which would jeopardize the structural integrity. With the increase in number of critical nodes in the network, the probability of one of these nodes failing in case of a random failure increases, reducing the resilience of the system. Furthermore, it also increases the number of sites for potential targeted attacks on the network.

The structural robustness of a network can be analyzed through studying the connectivity configurations and system performance of the network following a perturbation scenario of removal of one or more nodes and links due to random or targeted attacks. When complex networks are subject to random removal of nodes or links, i.e. a fraction f of the nodes are removed randomly, they can be analyzed as a case of infinite-dimensional percolation (21). Percolation theory indicates the presence of a critical probability p_c below which the network is composed of individual isolated clusters and above which there remains a giant cluster spanning the entire network. While the percolation theory is defined on a regular d-dimensional lattice, it meets random networks exactly in the infinite-dimensional limit ($d \to \infty$) of percolation (22). When f exceeds a certain threshold f_c i.e. the critical ratio of defragmentation, the network loses its large-scale connectivity and defragments. At a f value less than f_c, the network contains a connected cluster spanning the entire system, the size of which is proportional to the system size before perturbation. Exact solutions of f_c are available for two types of random networks – Cayley trees (23) and Erdös-Rényi random graph (24). For any graph f_c can be estimated from the following expression:

$$f_C = 1 - \frac{1}{\kappa_0 - 1} \qquad (4)$$

where, $\kappa_0 = \langle k_0^2 \rangle / \langle k_0 \rangle$ is computed from the pre-perturbed graph and $\langle k_0 \rangle$ is the average node degree of the graph. This metric provides gainful comprehension about the robustness of the network in the event of catastrophic natural events like earthquakes. However, water and energy distribution networks are unique in their structural characteristics. The most critical nodes in these networks are not necessarily the hubs, i.e. the nodes with the maximum connection or the most central ones; rather they are the most influential ones, like the source nodes and the links directly connected to the source nodes. This individuality necessitates differentiation between the structural vulnerability of the network which can be estimated by f_c and fault-tolerance of the network. The fault-tolerance of a network can be estimated by examining network properties that quantify the robustness and optimal connectivity of the network. Algebraic connectivity, λ_2, a measure of the robustness and connectedness of the network was first introduced by Fielder (25) and is defined as the second smallest eigenvalue of the normalized Laplacian matrix of a network. The Laplacian matrix of a graph G with n nodes is an n square matrix $L = D - A$, where $D = diag(d_i)$, d_i being the degree of node i. $A = (a_{ij})$ is the adjacency matrix of the graph, where $a_{ij}=1$ if there is a link between nodes i and j, 0 otherwise. The smallest eigenvalue of a Laplacian matrix is zero having a multiplicity equal to the value of the network's connected components. For $n \geq 2$, λ_2 always have a positive value for any connected graph (25). A larger value of λ_2 is an indicator of higher resistance offered by the network towards efforts to decouple the network. The larger the λ_2, the larger the number of node- or link-disjoint paths in the network, i.e. the graph remains fully connected despite the removal of nodes or links. Jamakovic and Uhlig (26) ran probabilistic failure simulation on all major classes of complex networks, which corroborates that λ_2 can be used as a measure of the robustness of complex networks.

Another metric pertinent to the particular scenario of water distribution systems is the meshedness coefficient (r_m)(27). For a distribution network with n nodes and m links, the number of independent cycles in the network is represented by $f = m - n + 1$ for a single source system and $f = m - n$ for a multi-source system and cannot exceed $3n - 6$ for any planar graph (8). r_m is defined as the ratio of the actual number of cycles in the network to the maximum possible number of cycles in the network (bounded by $2n - 5$).

$$r_m = \frac{f}{2n - 5} \tag{5}$$

The meshedness coefficient quantifies the density of cycles in the network and is a measure of path redundancy in the network. A higher value of r_m indicates a higher probability of two nodes remaining connected despite a link failure. This six metrics have been evaluated for the original and four alternate configurations of the Anytown network developed in this study and are shown in Table 1.

Table 1: Network metrics for the original and four alternate configurations of the AtN.

Network Metrics	***AtN***	***AtN^1***	***AtN^2***	***AtN^3***	***AtN^4***
Graph Diameter (d)	5.00	5.00	5.00	5.00	5.00
Characteristic Path Length (l)	1.24	1.31	1.32	1.34	1.26
Central-point dominance (c'_b)	0.28	0.12	0.14	0.09	0.10
Critical ratio of defragmentation (f_C)	0.63	0.63	0.61	0.58	0.62
Algebraic Connectivity (λ_2)	0.56	0.44	0.51	0.57	0.67
Meshedness Coefficient (r_m)	0.51	0.51	0.49	0.41	0.44

Multi-Criteria Analysis (MCA)

While the network metrics discussed in section 3.1 are pertinent to the structural organization and resilience of the water distribution system, they are discreet and hence challenging to compare across different alternatives. Multi-criteria Analysis (MCA) provides an attractive approach to unify all these metrics in a composite index of network resilience to provide the decision makers with a single numerical value to appraise the alternatives. The goal of this study was maximization of the resilience of the network based on its topology. The relative importance of each metric can be represented by assigning weights to each metric. The goal of an MCA model is to assess a finite set of decision options or alternative scenarios based on a set of evaluation criteria. An MCA model can be represented by an evaluation matrix (EM) X containing n decision options and m evaluation criteria (28).

$$X = \begin{pmatrix} x_{1,1} & \cdots & x_{n,1} \\ \vdots & \ddots & \vdots \\ x_{1,m} & \cdots & x_{n,m} \end{pmatrix} \tag{6}$$

$x_{i,j}$ represents the raw performance score of alternative *i* with respect to criterion *j*. to warranty an outcome of the MCA evaluation there needs to be at least two alternatives and two decision criteria, i.e. $n \geq 2$ and $m \geq 2$. The relative importance of each criterion is denoted by a one-dimensional weighing vector *W* which contains *m* weights, with w_j denoting the weight assigned to the j^{th} criterion.

$$W = w_1 \cdots w_n \tag{7}$$

The MCA evaluation aims to assign a utility score *u*, a single numerical measure of an alternative relative to the other alternatives; to each decision option by defining a utility function $u_i = f(X, W)$ where $U = \{u_1 \cdots u_n\}$ (29). In cases where there is discrimination or strict dominance, i.e. one alternative outperforms all others against all criteria (30), certain criteria or decision options need to be excluded from the MCA model. As the graph diameter for all the alternate scenarios were identical it was excluded in further analysis. In order to develop a utility score (u_i) for each alternative, the raw performance scores ($x_{i,j}$) for each criterion need to be transformed to a unit less value score ($v_{i,j}$). This study employs linear transformations to convert $x_{i,j}$ to $v_{i,j}$:

$$v_{i,j} = \frac{x_{i,j} - \min_{i=1}^{n}\left(x_{i,j}\right)}{\max_{i=1}^{n}\left(x_{i,j}\right) - \min_{i=1}^{n}\left(x_{i,j}\right)} \tag{8}$$

for a higher value of $x_{i,j}$ representing a better performance, and

$$v_{i,j} = \frac{\max_{i=1}^{n}\left(x_{i,j}\right) - x_{i,j}}{\max_{i=1}^{n}\left(x_{i,j}\right) - \min_{i=1}^{n}\left(x_{i,j}\right)} \tag{9}$$

for a lower value of $x_{i,j}$ representing a better performance, where $\max_{i=1}^{j}\left(x_{i,j}\right)$ is the maximum value of $x_{i,j}$ for $i = 1 \cdots n$ and $\min_{i=1}^{j}\left(x_{i,j}\right)$ is the minimum value of $x_{i,j}$ for $i = 1 \cdots n$. All the network metrics in consideration were converted to a unit less value score for the alternatives with the goal of maximizing the network resilience and efficiency.

Analytic Hierarchy Process (AHP) was used to assign weights to the network metrics. AHP essentially arranges the criteria in a hierarchical manner to satiate the goal or objective of the MCA (31). In AHP the criteria are compared pair wise based on a semantic scale of 1-9, which is defined to indicate how many times more important or dominant one element is over another element with respect to the criterion or property with respect to which they are compared to construct a $n \times n$ matrix, where n is the number of criterion being compared (32). All the alternatives were compared pair-wise with respect to the objective, which was to maximize the network resilience. For example, in Resilience Scenario, it is assumed that algebraic connectivity (λ_2) is strongly preferred (4 times) over characteristic length (l) in maximizing network resilience. The pair wise comparison matrices for the two scenarios are shown in Table 2.

The normalized principal Eigen vector of the pair wise comparison matrix provides the weighting matrix for the criteria. One important attribute of the decision making process in the AHP is the consistency of the estimator. In the instance of absolute consistence, the principal eigenvalue (λ_{max}) would be equal to *n*. For general cases, absolute consistence is unrealistic to be achieved (33). To address this issue, Saaty (31) proposed the consistency index (CI) as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1} \qquad (10)$$

In the instance of absolute consistence, $CI=0$ and $\lambda_{max}=n$ (31). For all other cases, the level of inconsistency can be measured by the consistency ratio (CR), defined as

$$CR = \frac{CI}{RI} \qquad (11)$$

where, *RI* or the random index, is the average value of CI for randomly generated reciprocal matrices using the Saaty scale (scale of 1-9). The weighting vector obtained is regarded as consistent *iff* CR<10%. Weighting vectors obtained for the two scenarios are given in Table 2.

Table 2: Pair wise comparison matrix and the weights obtained through AHP

	l	c'_b	f_C	λ_2	r_m		**Weights**
l	1	⅓	½	¼	½		8.2%
c'_b	3	1	2	⅓	2		20.7%
f_C	2	½	1	½	1	⇨	15.8%
λ_2	4	3	2	1	2		39.5%
r_m	2	½	1	½	1		15.8%

The MCA model combines the weights obtained from the AHP with the values to determine the overall utility of the alternative. This study uses the Weighted Summation (WS) approach, arguably the simplest and most widely used technique for this purpose. In the WS approach, the criteria are morphed onto a commensurate scale of 0 to 1, with 1 representing the best performance. The utility score for each alternative is determined using Eq. 9.

$$u_i = \sum_{j=1}^{m} v_{i,j} w_j \qquad (12)$$

where, $\sum_{j=1}^{m} w_j = 1$ and $0 < w_j \leq 1$.

RESULTS AND DISCUSSION

The utility scores obtained for each criterion for the alternate designs of AtN are presented in Figure 3. Alternate AnT^4 performs the best in amongst all other alternatives in terms of resiliency maximization. While the original configuration outperforms AtN^4 in a few of the metrics, AtN^4 owes its higher performance due to a relative low value of the central-point dominance (c'_b) and a high value of algebraic connectivity (λ_2). The original configuration of AtN has the highest c'_b value of 0.28. Since the alternate configurations were developed with the goal to maximize the network resilience, all of them have significantly lower c'_b value. For example, AnT^4 has a c'_b value 64% lower than the original configuration. Lowering the c'_b value is critical in order to increase the resilience of the system. Water distribution systems have certain critical vulnerable nodes like the source nodes, failure of which can cripple the entire network. Designing a system which adds more critical nodes (in terms of stability) to the system proportionally would increase its vulnerability to random failures and would be an imprudent decision to make from the resilient perspective.

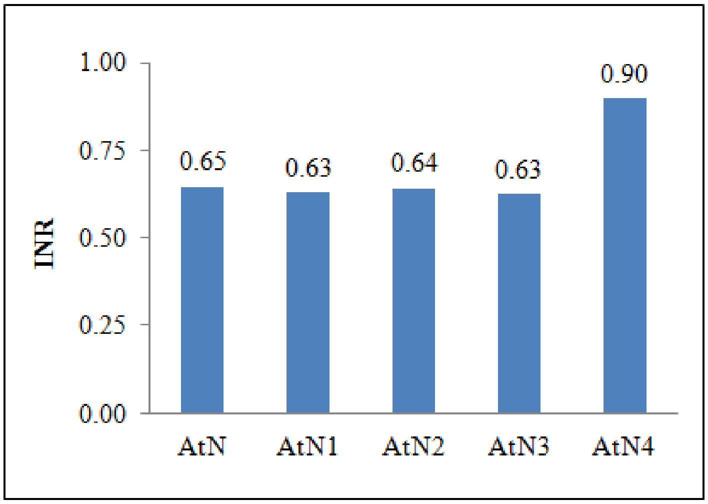

Figure 3: INR values obtained for the alternate AtN configurations

The algebraic connectivity (λ_2) has been assigned the maximum weight (~40%). While critical ratio of defragmentation (f_c), meshedness coefficient (r_m) and algebraic connectivity all are related to the resilience of the network, f_c and r_m are directly proportional to the number of links in the network. Arguably, the stability and redundancy in a network can be increased by increasing the number of connections, i.e. the links, between the nodes. However, it also involves larger investment in terms of capital cost, material and energy which is not justifiable from the perspective of sustainability. Conversely, λ_2 is solely dependent on the layout of the network not related to the number of pies. Thus it offers a unique perception on ways to increase the robustness and redundancy of a network without increased material and energy investment.

SUMMARY AND CONCLUSION

This paper presented a novel approach to quantify resilience of urban water distribution systems using a complex network approach. The systems were analyzed as planar graphs represented by links and nodes of interconnected components. Six attributes of network topology, ① graph diameter *(d)*, ② characteristic path-length *(l)*, ③ central-point dominance *(c'_b)*, ④ critical ratio of defragmentation *(f_c)*, ⑤ algebraic connectivity *(λ_2)*, and ⑥ meshedness coefficient *(r_m)* were identified which address two of the major criticalities of distribution systems efficiency and resilience. An MCA model was developed using the weighted summation approach to choose the best alternative design for the hypothetical Anytown Network (AtN). Two alternate scenarios were considered, resilience maximization and efficiency maximization, to assess if there is a potential trade-off between resilience and efficiency based on the network topology.

Based on the results, the following conclusions can be drawn:

1. Resilience, or the capacity of a system to absorb and perform under perturbation, can be used as an effective comprehensive indicator of sustainability for urban water systems.
2. The index of resilience (INR) developed in this study provides significant comprehension about how network topology affects the resilience of water distribution systems.
3. AnT^4 is the most resilient alternative based on the network topology.

4. The original configuration has the highest value for c'_b, which adds nodes to the system which are critical in order to maintain the large-scale integrity of the network in case of random or targeted attacks.
5. Algebraic connectivity provides a measure of the robustness and redundancy in the network structure being independent of the number of links present in the network.

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